Fabrication of Very Weakly and Weakly Magnetized Microstrip Circulators

Joseph Helszajn, *Fellow, IEEE*

***Abstract*—The synthesis of practical microstrip circulators in terms of a microwave specification remains in practice a difficult problem. If a very weakly magnetized model is adopted for its gyrator circuit, this task should in principle be (once the quality factor of the resonator is stipulated) a simple endeavor provided the substrate thickness can be varied. If it is not possible to do so, it is necessary to either alter the resonator shape or the minimum ripple level of the microwave specification or vary both quantities. This paper demonstrates that the very weakly magnetized model of a junction circulator using either a side or an end-coupled triangular resonator is more restricted than may be at first supposed. A weakly magnetized solution for the design of such junction circulators which bridges the interval between the very weakly and the moderately magnetized regimes is the main contribution of this paper.**

***Index Terms*—Circulators, devices, ferrites, nonreciprocal.**

I. Introduction

THE DESIGN of quarter-wave coupled microstrip circulators still depends on experimental cut-and-try procedures. Some early experimental examples of this sort of hardware using disk resonators are to be found in [1]–[7]. In order to have a synthesis procedure, it is necessary to reconcile the physical details of the resonator, its gyrotropy, and the network one. One class of circulators for which a solution is possible is that based on the classic very weakly magnetized model of a disk resonator [8]–[10]. In this arrangement, the susceptance slope parameter of the gyrator circuit is dependent upon the substrate thickness, the quality factor, or gain bandwidth on the gyrotropy. The gyrator conductance is in this sort of problem the dependent quantity and is met once the other two quantities are specified. The descriptions of stripline circulators using very weakly magnetized triangular resonators coupled midway along its sides or at its corners are separately available [11]. While the quality factor of each of these resonators is of the same order, small differences in this quantity are relevant; each, however, has a quite different value of susceptance slope parameter. Specifically, the susceptance slope parameter of the side-coupled triangular resonator is three times that of a simple disk resonator, whereas that of the apex fed one is one-third that of the disk resonator [11]. Each circuit is, therefore, associated with a quite different microwave solution. Since the substrate thickness is often specified by equipment requirements rather than by the circulator design, any practical design procedure must be based on the ability to vary the resonator shape. A complete description of a circulator using a very weakly magnetized regular hexagonal resonator is also available [12]. Suffice it to note that its gyrator circuit is not very different from that associated with a simple disk resonator. Some possible resonator shapes met in the design of three-port circulators are indicated in Fig. 1.

The sort of specification that can in practice be realized with this class of circulation solutions depends of course on what is meant by a very weakly magnetized resonator. For the purpose of this paper, a resonator is either very weakly magnetized, weakly magnetized, moderately magnetized, or strongly magnetized. The very weakly magnetized model of a junction circulator using a disk gyromagnetic resonator already shows significant deterioration at a gyrotropy (κ) equal to 0.25 and that of the weakly magnetized regime breaks down at $\kappa = 0.30$. This paper indicates that the corresponding values for a triangular resonator are $\kappa = 0.25$ and 0.35. The construction of a weakly magnetized model of a junction circulator using a triangular resonator proceeds in the same fashion as that previously employed to describe that using a disk geometry. It amounts to expanding the quality factor and the susceptance slope parameter about the very weakly magnetized closed-form descriptions in terms of a suitable polynomial in the gyrotropy over the interval for which the gyrator conductance obeys the weakly magnetized model. While the gain–bandwidth product of this class of solutions is restricted by the gyrotropy of the problem region, it still provides practical commercial solutions. Of course, different conclusions are possible if a moderately or a strongly magnetized resonator is utilized [23]–[26].

The network problem for this class of device is separately well understood; it readily yields a relationship between the network specification [$S(\min)$, $S(\max)$, and bandwidth)] and the one-port description of the junction circulator (susceptance slope parameter and gyrator conductance, and by definition, loaded-Q factor) [9]–[17]. The use of the alternate line transformer deserves special mention in the design of microstrip circulators [16]. A feature of this sort of general network problem is that while its maximum ripple level in the passband and its bandwidth specification are essentially fixed by the Q factor of the load, the value of its minimum ripple level may be used to control the level of its susceptance slope parameter. Since the equipment maker is seldom interested in this latter quantity, it may in practice be used to absorb any uncertainty in the precise characterization of the absolute value of the susceptance slope parameter of the gyrator network.

This paper includes the description of one commercial quarter-wave coupled microstrip circulator based on a side-

Manuscript received June 10, 1995; revised February 13, 1998.
The author is with the Department of Computing and Electrical Engineering, Heriot-Watt University, Edinburgh, Scotland EH14 4AS U.K.
Publisher Item Identifier S 0018-9480(98)03157-3.

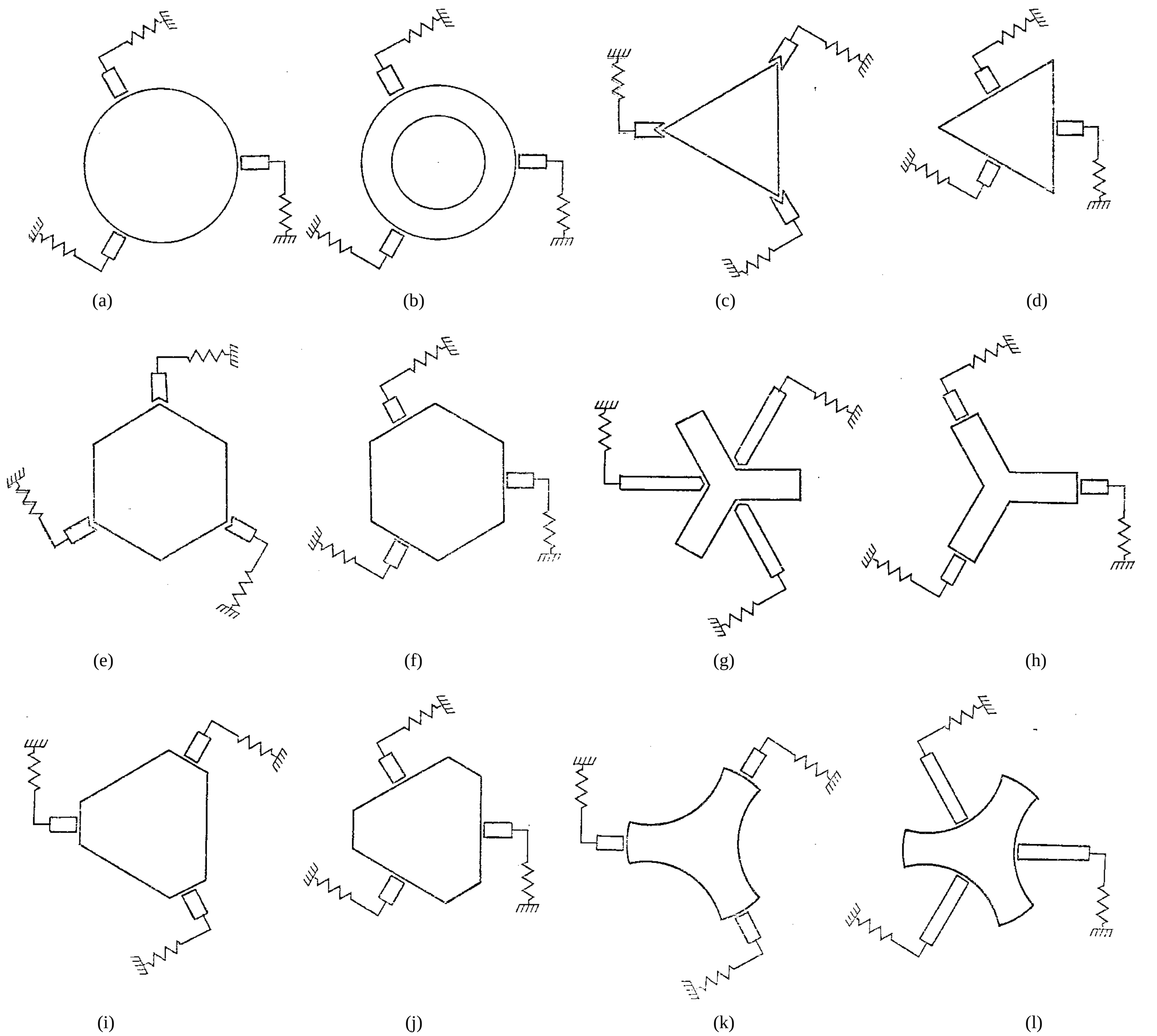


Fig. 1. Schematic diagram of planar resonators for use in planar junction circulators.

coupled triangular resonator. It also reviews the design features of two published arrangements. If an idealized template is superimposed on the frequency responses of these devices, it should in principle be possible to deduce the element values of the corresponding complex gyrator circuits. Unfortunately, while the specifications of each of these devices were of commercial quality, the detailed specifications were not, in the absence of Smith chart data, sufficiently robust to draw any firm conclusions other than to note that the calculated effective Q factor was in every instance lower than that inferred by the experimented gain–bandwidth product.

II. Parallel-Plate Waveguide Model of Microstrip Circulators

The usual approach to the design of microstrip passive circuits and circulators using weakly magnetized resonators is to replace the problem region with imperfect magnetic sidewalls by an equivalent waveguide model, which is the method adopted here [30]–[33]. Fig. 2(a)–(c) indicates possible equivalencies for microstrip circulators using planar circular and triangular resonators. The concepts entering into this sort of model are well established and need not be dealt with [30]. Once any design is complete in terms of the equivalent parallel-plate waveguide approximation, it is necessary to invoke the relationship between the actual and effective parameters of the problem region in [30]–[33].

The effective quantities entering into the description of a planar triangular resonator with open walls are defined by introducing an effective inscribed radius (r) in the calculation of the equilateral triangular resonator. No justification is sought for this decision other that it permits some initial engineering parts to be laid out on the basis of an equivalent waveguide model of the problem region. The agreement between this empirical model and the graphical data in [11] and [32] is, however, acceptable for design.

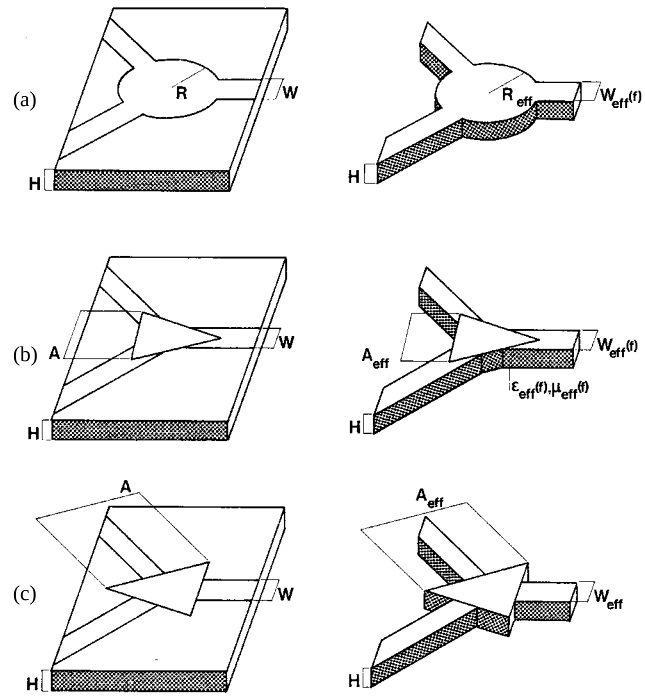


Fig. 2. Equivalence between microstrip resonators and parallel waveguide circuits.

Two matters of concern in the design of such circuits are that if the fringing fields on a typical contour are excessive then it becomes difficult to preserve the definitions of the coupling angle at the terminals of the resonator and its shape. The effect on the resonator shape is particularly worrisome in that it may no longer have any resemblance to the actual one. This situation may be understood by recognizing that the effective sidewalls of a high-impedance strip are displaced further out from the physical ones than is the case of those of a low-impedance strip. This sort of effect would readily distort the equilateral resonator in Fig. 1(c) or (d), say, into something more like the irregular hexagonal resonator with concave sidewalls illustrated in Fig. 1(k) or (l). It is also necessary to ensure that fringing effects do not corrupt the complex gyrator description of the circuit. One way to partially avoid both difficulties is to impose a lower bound on the aspect ratio (r/H) of the resonator [21], [22]. Some separate remarks about the influence of the aspect ratio of the resonator region upon the uniformity of the magnetization has been described in [40]. While one arbitrary bound on this quantity has been mentioned [22], a still more restrictive one is recommended here:

$$\frac{r}{H} \geq 5. \tag{1}$$

r either represents the inscribed radius defined by the triangular resonator

$$r = \frac{A}{2\sqrt{3}} \tag{2a}$$

or the actual radius of the disk

$$r = R. \tag{2b}$$

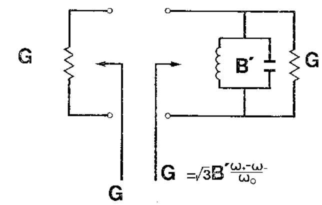


Fig. 3. One-port complex gyrator circuit of three-port junction circulator.

TABLE I
DESIGN DATA FOR VERY WEAKLY MAGNETIZED DISK RESONATOR
$k_{\text{eff}}(f)R_{\text{eff}}(f) = 1.84,\ 2\delta_0 = (f_2 - f_1)/f_0,\ \zeta_{\text{eff}} = \sqrt{\frac{\varepsilon_{f,\text{eff}}(f)}{\mu_{d,\text{eff}}(f)}}$ [8]

Network Variables	Magnetic Variables	Physical Variables	Frequency Variables
B'	—	$\frac{0.740\zeta_0(k_{\text{eff}}(f)R_{\text{eff}}(f))^2}{\mu_{e,\text{eff}}(f)k_0H}$	—
G	$[\frac{2\sqrt{3}B'}{(k_{\text{eff}}(f)R_{\text{eff}}(f))^2-1}](\frac{\kappa}{\mu})$	$\zeta_{\text{eff}}\zeta_0\frac{W_{\text{eff}}(f)}{H}$	$\sqrt{3}B'(\frac{\omega_+-\omega_-}{\omega_0})$
Q_L(eff)	$[\frac{(k_{\text{eff}}(f)R_{\text{eff}}(f))^2-1}{2\sqrt{3}}](\frac{\mu}{\kappa})$	$\frac{0.740\zeta_0(k_{\text{eff}}(f)R_{\text{eff}}(f))^2}{G\mu_{e,\text{eff}}(f)k_0H}$	$\left[\sqrt{3}(\frac{\omega_+-\omega_-}{\omega_0})\right]^{-1}$

One possible definition of the magnetic filling factor q_m entering into the description of the effective quantities is defined by [33]

$$q_m \approx \frac{1}{2}\left\{1 + \left[1 + \frac{5H}{r}\right]^{-0.55}\right\}. \tag{3}$$

q_m is the filling factor of a strip of thickness H whose width (W) equals $2r$.

A unique feature of a junction circulator using a very weakly or weakly magnetized resonator (upon which the design procedure of this paper rests) is that its complex gyrator circuit is essentially independent of the coupling angle at its ports. This means that the design of the transformer circuit does not perturb the junction in any way and that its adjustment may be dealt with by the appropriate parallel-plate model of a microstrip line on a demagnetized ferrite substrate [33].

III. VERY WEAKLY MAGNETIZED PROBLEM REGION

The design of a very weakly magnetized microstrip circulator using a disk or a triangular resonator proceeds with the aid of its equivalent waveguide model and a knowledge of the corresponding design procedure for stripline circulators. If the in-phase eigennetwork of the device may be idealized by a frequency-independent short-circuit boundary condition at its terminals then its complex gyrator circuit may be realized in the form indicated in Fig. 3. In this circuit, G is the absolute conductance at the terminals of the junction and also at the output microstrip line, B' is the absolute susceptance slope parameter, and Q_L is the loaded Q factor. Tables I–III summarizes some classic design equations for the realization of microstrip circulators using weakly magnetized disk and triangular resonators [3], [8], [10], [11].

The first column in these tables gives the susceptance slope parameter, gyrator conductance and loaded Q factor at the terminals of the device. The second column defines the ratio

TABLE II
DESIGN DATA FOR VERY WEAKLY MAGNETIZED TRIANGULAR RESONATOR COUPLED AT CORNERS ($k_{\text{eff}}(f)A_{\text{eff}}(f) = 4\pi/3,\ 2\delta_0 = (f_2 - f_1)/f_0$) [11]

Network Variables	Magnetic Variables	Physical Variables	Frequency Variables
B'	——	$\frac{0.0475\zeta_0(k_{\text{eff}}(f)A_{\text{eff}}(f))^2}{\mu_{e,\text{eff}}(f)k_0H}$	——
G	$[\frac{3B'}{\pi}](\frac{\kappa}{\mu})_{\text{eff}}$	$\zeta_{\text{eff}}\zeta_0\frac{W_{\text{eff}}(f)}{H}$	$\sqrt{3}B'(\frac{\omega_+-\omega_-}{\omega_0})$
Q_L(eff)	$\frac{\pi}{3}(\frac{\mu}{\kappa})_{\text{eff}}$	$\frac{0.0475\zeta_0(k_{\text{eff}}(f)A_{\text{eff}}(f))^2}{G\mu_{e,\text{eff}}(f)k_0H}$	$\left[\sqrt{3}(\frac{\omega_+-\omega_-}{\omega_0})\right]^{-1}$

TABLE III
DESIGN DATA FOR VERY WEAKLY MAGNETIZED TRIANGULAR RESONATOR COUPLED MIDWAY ALONG SIDE DIMENSIONS ($k_{\text{eff}}(f)A_{\text{eff}}(f) = 4\pi/3,\ 2\delta_0 = (f_2 - f_1)/f_0$) [11]

Network Variables	Magnetic Variables	Physical Variables	Frequency Variables
B'	——	$\frac{0.4275\zeta_0(k_{\text{eff}}(f)A_{\text{eff}}(f))^2}{\mu_{\text{eff}}(f)k_0H}$	——
G	$[\frac{3B'}{\pi}](\frac{\kappa}{\mu})_{\text{eff}}$	$\zeta_{\text{eff}}\zeta_0\frac{W_{\text{eff}}(f)}{H}$	$\sqrt{3}B'(\frac{\omega_+-\omega_-}{\omega_0})$
Q_L(eff)	$\frac{\pi}{3}(\frac{\mu}{\kappa})_{\text{eff}}$	$\frac{0.4275\zeta_0(k_{\text{eff}}(f)A_{\text{eff}}(f))^2}{G\mu_{\text{eff}}(f)k_0H}$	$\left[\sqrt{3}(\frac{\omega_+-\omega_-}{\omega_0})\right]^{-1}$

of the entries of the tensor permeability κ/μ of the magnetized resonator in terms of the loaded Q factor (or in terms of the gyrator conductance) and susceptance slope parameter of the device.

Once the magnetic variables of the resonator are fixed, its linear dimensions may be calculated using the entries in the third column in Tables I–III. The radius of the circular disk resonator or the side dimension of the triangular one is obtained from the respective cutoff numbers. The substrate thickness H is then obtained from the specification of the susceptance slope parameter. The calculation of $W_{\text{eff}}(f)$ completes the design and ensures that the network, magnetic, and physical variables in Tables I–III are compatible. If the specifications of the device are incompatible with the substrate thickness H, or the width $W_{\text{eff}}(f)$ of the connecting lines is incompatible with the resonator area, the design must be repeated with a less severe specification.

The last column in these tables gives the description of the gyrator conductance and loaded Q factor in terms of the split frequencies of the two counter-rotating modes of the magnetized resonator. It is of note that the quantities in Tables I–III are related through the following relationship [8], [9], [34]:

$$G = \sqrt{3}B'\left(\frac{\omega_+ - \omega_-}{\omega_0}\right). \tag{4}$$

This equation indicates that G is proportional to the product of the difference between the two split frequencies of the counter-rotating modes of the magnetized junction and the susceptance slope parameter of the junction. This is a general result for any very weakly magnetized junction circulator.

Strictly speaking, it is also necessary to introduce the notion of an effective gyrotropy in the description of the quality factor of the resonator. One semiempirical formulation is

$$\left(\frac{\kappa}{\mu}\right)_{\text{eff}} \approx q_m\left(\frac{\kappa}{\mu}\right). \tag{5}$$

The following effective permeability $\mu_{e,\text{eff}}$ is separately given:

$$\mu_{e,\text{eff}} = \frac{\mu^2 - (q_m\kappa)^2}{\mu} \tag{6}$$

where q_m is the filling factor defined in (3). It is assumed here that $(\mu)_{\text{eff}} = \mu$ and $(\kappa)_{\text{eff}} = q_m\kappa$.

In order to have a trustworthy synthesis procedure for the design of these devices, it is necessary to place an upper bound on the magnetic variables (splitting) over which the entries in Tables I–III may be used with confidence. A very weakly magnetized junction is defined for the purpose of this paper as one for which the split counter-rotating eigennetworks or eigenvalues may be described by single split poles and for which the in-phase one may be represented by a frequency-independent electric wall at the terminals of the resonator [26]. One definition of a weakly magnetized junction is, therefore, the verification of this condition. Another, which is in keeping with its very weakly magnetized model, is to ensure that its Q factor is independent of the coupling angle at the resonator terminals. Still another test is to ensure that the susceptance slope parameter is independent of both its gyrotropy and its coupling angle. An investigation of this problem in the case of a circulator using a disk resonator indicates that its very weakly magnetized model already displays significant deterioration in the closed-form description of its complex gyrator circuit when the value of its gyrotropy (κ) is as little as 0.25. [9]. The gyrotropy in the case of a junction using a side-coupled triangle is also restricted to about 0.25 [37]. While such values of gyrotropy places some restriction on the gain–bandwidth product of this sort of solution, it is nevertheless adequate in the design of practical specifications. The moderately magnetized region actually produces a more attractive gain–bandwidth product, but at the cost of a more difficult adjustment procedure.

IV. WEAKLY MAGNETIZED PROBLEM REGION

One approach that has been used in order to extend the description of the very weakly magnetized description of a junction circulator using a disk resonator has been to introduce suitable correction terms based on some exact calculations [9]. It will now be extended to the description of a junction using a weakly magnetized triangular resonator by making use of some calculations on a side-coupled triangular resonator based on the contour integral method [37]. Scrutiny of this solution indicates that its Q factor and susceptance slope parameter deteriorate more rapidly than its gyrator conductance. This feature has been separately observed in [35]. One means of dealing with this problem is to write the actual absolute susceptance slope parameter $B'(\kappa/\mu)$ in terms of a modified Q factor $Q_L(\kappa/\mu)$ and the very weakly magnetized description of the gyrator conductance G as

$$B'(\kappa/\mu) = Q_L(\kappa/\mu)G, \qquad 0 \le \kappa \le 0.35 \tag{7}$$

or

$$B'(\kappa/\mu) = Q_L(\kappa/\mu)\left[\frac{B'(0)}{Q_L}\right], \qquad 0 \le \kappa \le 0.35. \quad (8)$$

G is the absolute value of the gyrator conductance in Tables I–III and $Q_L(\kappa/\mu)$ is given by [10]

$$Q_L(\text{eff}) \approx 0.689\left(\frac{\mu}{\kappa}\right) + \left[0.046 - 2.632\left(\frac{\kappa}{\mu}\right) + 3.551\left(\frac{\kappa}{\mu}\right)^2\right]$$
$$0 \le \left(\frac{\kappa}{\mu}\right) \le 0.30 \quad 0.10 \le \Psi_{\text{eff}}(f) \le 0.50. \quad (9)$$

One suitable polynomial expansion for the quality factor $Q_L(\text{eff})$ of a weakly magnetized microstrip circulator using a disk resonator is obtained by introducing the filling factor q_m in the stripline relationship

$$Q_L(\text{eff}) \approx 0.689\left(\frac{1}{q_m}\right)\left(\frac{\mu}{\kappa}\right) + \left[0.046 - 2.632\, q_m\left(\frac{\kappa}{\mu}\right) + 3.551 q_m^2\left(\frac{\kappa}{\mu}\right)^2\right]$$
$$0 \le q_m\left(\frac{\kappa}{\mu}\right) \le 0.30 \quad 0.10 \le \Psi_{\text{eff}}(f) \le 0.50. \quad (10)$$

The one obtained here in the case of a triangular one based on the data in [37] is

$$Q_L(\text{eff}) = \frac{\pi}{3}\left(\frac{1}{q_m}\right)\left(\frac{\mu}{\kappa}\right) + \left[1.486 - 12.71 q_m\left(\frac{\kappa}{\mu}\right) + 19.74 q_m^2\left(\frac{\kappa}{\mu}\right)^2\right]$$
$$0 \le q_m\left(\frac{\kappa}{\mu}\right) \le 0.35 \quad 0.52 \le \Psi_{\text{eff}}(f) \le 1.20 \quad (11)$$

where

$$\sin\,\Psi_{\text{eff}}(f) = \frac{W_{\text{eff}}(f)}{2R_{\text{eff}}(f)} \quad (12)$$

and

$$\sin\,\Psi_{\text{eff}}(f) = \frac{W_{\text{eff}}(f)}{2r_{\text{eff}}(f)} \quad (13)$$

respectively.

The values of Q factor bracketed by the above two relationships are valid for both the very weakly and weakly magnetized resonators specified in this paper. Since it is usually desirable in any design to maximize the gain–bandwidth product of the load, the only values of gyrotropy that are of any interest are those associated with the upper bounds defined by these intervals. The corresponding lower bounds on the Q factors of the two arrangements are 1.86 and 2.42, respectively. These results suggest (other factors being equal) that the gain–bandwidth product of a junction circulator employing a circular disk resonator is more attractive for the design of commercial devices than one using a triangular one. One feature of this solution is that the Q factor is independent of the coupling angle.

A separate scrutiny of the first circulation condition suggests that it also needs some modification. One possible explanation of this sort of discrepancy is that, strictly speaking, the resonant frequency displayed by a junction circulator coincides with that at which the reflection eigenvalues of the in-phase and degenerate counter-rotating eigennetworks are out of phase rather than with that of the counter-rotating ones. This means that the actual boundary condition that needs to be satisfied is

$$X_{\text{in}} = 0 \quad (14)$$

instead of

$$X_1 = 0. \quad (15)$$

X_{in} represents the imaginary part of the impedance of the complex gyrator circuit for the case for which the reactance of the in-phase eigennetwork is different from zero, X_1 represent the same quantity when it is equal to zero.

The solution to the former so-called first circulation condition has, in the case of a triangular resonator, in fact been solved using the contour integral method [37]. It suggests in keeping with the experimental data achieved here that

$$k_{\text{eff}}(f)A_{\text{eff}}(f) \le \left(\frac{4\pi}{3}\right) \quad (16)$$

where

$$k_{\text{eff}}(f) = k_0\sqrt{\varepsilon_{f,\,\text{eff}}(f)\mu_{e,\,\text{eff}}(f)}. \quad (17)$$

The equivalent result for a disk is also readily available in the literature. If these considerations are embodied in the design, then the agreement between design and practice is good.

The second circulation condition is met by satisfying the appropriate entries specified by (7) and (8).

V. Experimental Evaluation of Complex Gyrator Circuits

The split frequencies of one microstrip junction using a triangular resonator with one value of magnetization is indicated in Fig. 4. Its Q factor and also those of a similar resonator, but with a different value of magnetization coupled at each of its two possible triplets of terminals, are summarized in Fig. 5. The relationship between the Q factor of this sort of circuit and the gyrotropy of the resonator is self evident. Fig. 6 shows some results for a disk resonator for three different materials which may be used to verify the relationships between the magnetic and frequency variables tabulated in columns two and four in Tables I–III. Fig. 7 indicates that the Q factor of a disk resonator is for a given gyrotropy lower than that of the corresponding triangular geometry in keeping with theory.

The correlation between theory and experiment in the case of the loaded Q factor of the disk geometry has been dealt with previously and will not be discussed further. That of a triangular resonator is addressed here. To cater for saturation effects in a magnetized substrate, an initial value of loaded Q factor is defined [9]. This quantity is constructed by extending the angle suspended by the split frequencies at the origin to the value of applied direct magnetic field at saturation. This field coincides with that for which the internal direct magnetic

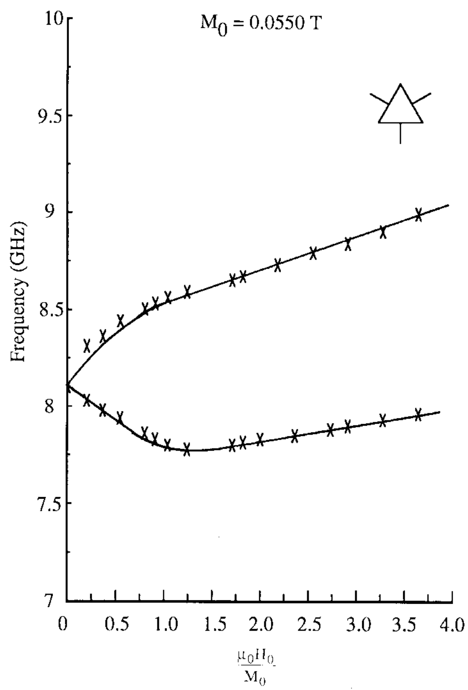


Fig. 4. Split frequencies versus applied direct magnetic field for loosely coupled side-coupled triangular resonator on garnet substrate ($p = 0.190$).

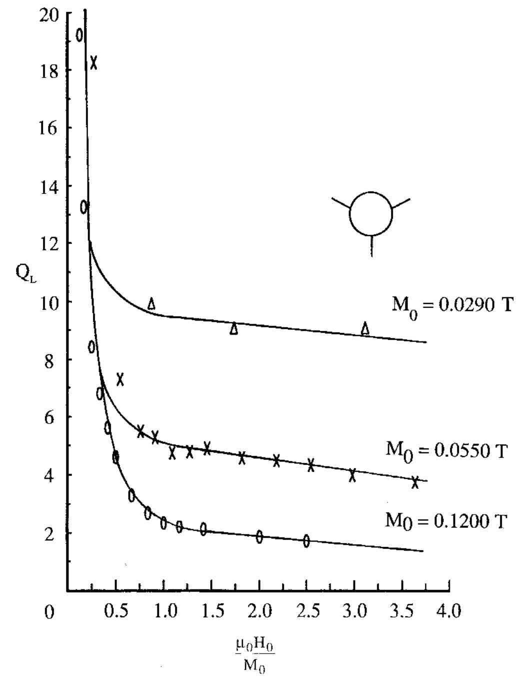


Fig. 6. Q factor versus applied direct magnetic field for loosely coupled disk resonator on a different garnet substrates ($p = 0.090$, 0.174, and 0.388).

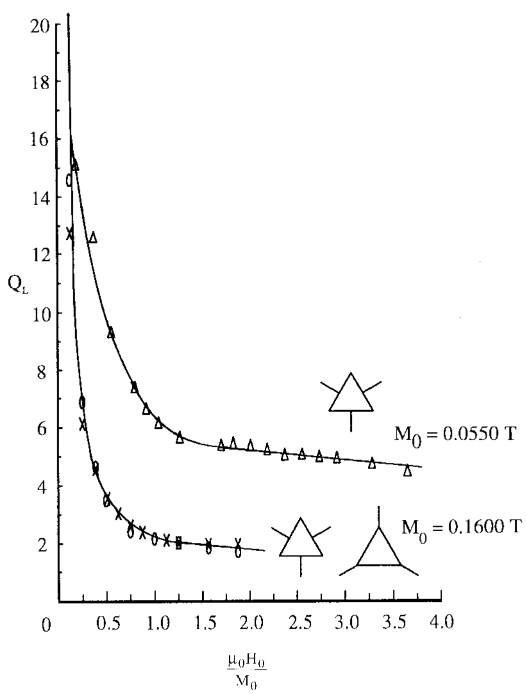


Fig. 5. Q factor versus applied direct magnetic field for loosely coupled triangular resonator on different garnet substrates ($p = 0.190$ and 0.533).

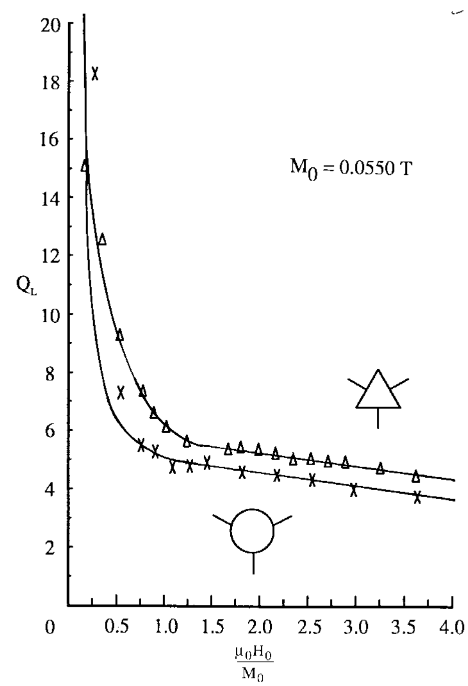


Fig. 7. Comparison between Q factors of disk and triangular resonators.

field (H_i) is equal to zero

$$H_i = 0 \tag{18}$$

where

$$H_i = H_0 - N_z\left(\frac{M_0}{\mu_0}\right). \tag{19}$$

M_0 is the saturation magnetization (T), γ is the gyromagnetic ratio [2.21 × 105 (rad/s)/(A/m)], ω is the radian frequency (rad/s), μ_0 is the free-space permeability ($4\pi \times 10^{-7}$ H/m), N_z is the demagnetizing factor along the axis of the resonator, and H_0 is the direct magnetic field (A/m).

If the material is not saturated then it is necessary to replace q_m in the description of $Q_L(\text{eff})$ by

$$q_m\left(\frac{M}{M_0}\right) \tag{20}$$

where M is the actual magnetization.

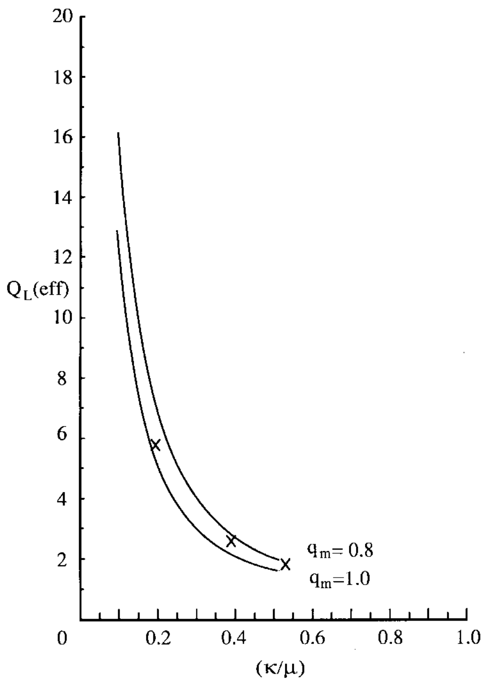


Fig. 8. Comparisons between Q factors based on closed-form numerical method and experimental values of a triangular resonator versus gyrotropy $(r/H = 2.73)$.

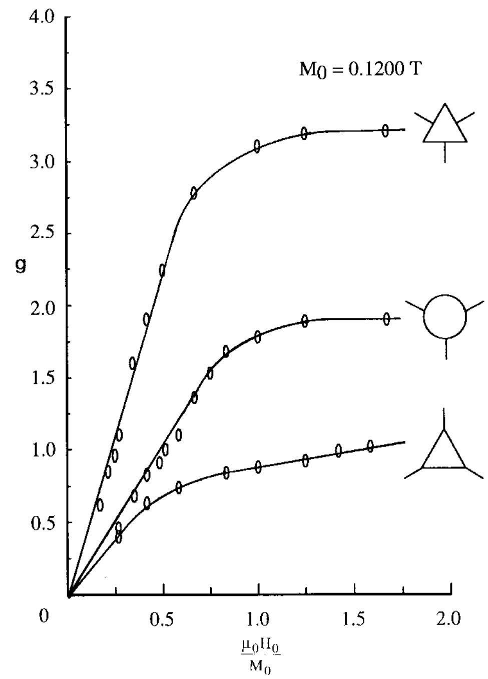


Fig. 9. Gyrator conductance versus applied direct magnetic field for disk, apex, and sidewall coupled triangular resonators on garnet substrate $(p = 0.388)$.

The relationship between the gyrotropy and effective Q factor $Q_L(\text{eff})$ of a microstrip resonator using a triangular resonator is illustrated in Fig. 8 for $q_m = 0.80$ and 1.0. Some experimental results which rely on a definition of $Q_L^i(\text{eff})$ are separately superimposed on this illustration. While these Q-factor curves appear at first sight well behaved, this is not in itself sufficient since it is also necessary to ensure that the susceptance slope parameter is independent of the gyrotropy over the same interval.

Fig. 9 depicts some experimental data at about 9 GHz on the gyrator conductance of junction circulators using disk, side, and apex coupled triangular resonators printed on a 0.635-mm substrate with a saturation magnetization (M_0) equal to 0.1200 T. This sort of data is obtained by making use of the relationship between the midband return loss of a magnetized junction and its gyrator conductance with the two output ports terminated by 50-Ω loads [34]. It is evident from this result that the gyrator conductance of the three arrangements under consideration are not in the ratio 1:3:9 of the respective values of the susceptance slope parameters in Tables I–III. One obvious reason for these discrepancies is that the values of the Q factor of disk and triangular resonators are slightly different. Another is that the values of the aspect ratios of the resonators employed to obtain this data $(r/H \approx 2.73)$ were somewhat lower than that stipulated earlier in order to adequately reproduce the resonator shape of the problem region.

A scrutiny of the network problem readily indicates that none of the complex gyrator circuits displayed by these geometries are (at first sight) appropriate at 9 GHz for the design of quarter-wave coupled devices on a 0.635-mm substrate. Some experimental hardware fabricated in this paper is illustrated in Fig. 10.

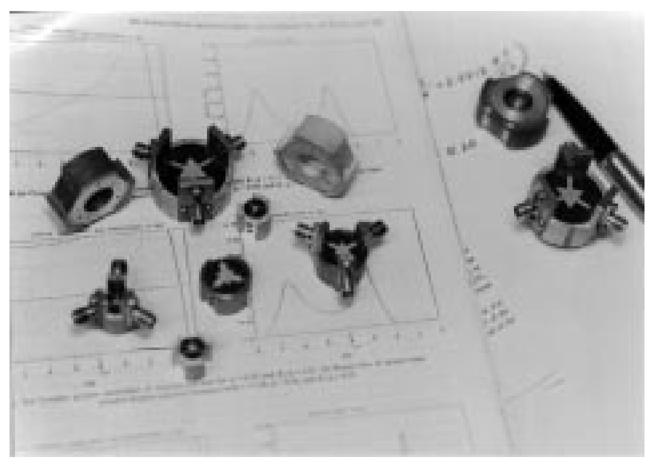


Fig. 10. Photograph of microstrip circuits.

VI. Synthesis of Quarter-Wave Coupled-Junction Circulators

The synthesis of quarter-wave coupled microstrip circulators in terms of a microwave specification is a classic result [8], [13]–[17].[1] Standard tables for the design of this type of circuit are given in [14] in the case where $S(\min)$ equals unity and in [15] for the more general case when $S(\min)$ is different from unity. Once the circulator specifications are specified, the loaded Q factor is evaluated and its value is used to check whether it is compatible with the very weakly or weakly magnetized models employed in this paper. If it is, the gyrator conductance and modified susceptance slope parameter

[1] It is further assumed that the network variables apply provided $(\omega_+ - \omega_-)/\omega_0 \geq (\omega_2 - \omega_1)/\omega_0$, $\omega_{1,2}$ and ω_0 specify the band edges and center frequencies.

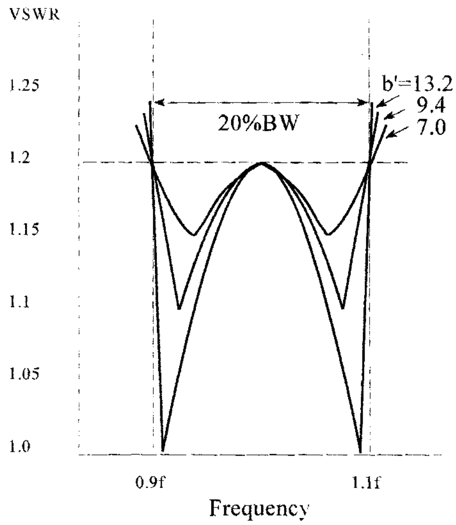


Fig. 11. Frequency responses of quarter-wave coupled circulators showing effect of varying susceptance slope parameter (b').

may be evaluated. The design procedure then continues by selecting the resonator shape and evaluating the substrate thickness and width of the connecting lines. In this design problem, the Q factor and the susceptance slope parameter are the independent variables and the gyrator conductance is the dependent one. The Q factor is fixed by the gyrotropy of the gyromagnetic resonator, and the susceptance slope parameter by the resonator shape and substrate thickness and, to some extent, the gyrotropy. A feature of this sort of problem is that varying the value of the minimum ripple level or VSWR in the passband leaves the maximum ripple level or VSWR and the passband specification unchanged, but produces relative large variations in the absolute value of the susceptance slope parameter of the load. Fig. 11 shows a typical situation. A detailed scrutiny of the network problem indicates that a wide family of practical specifications may in fact be realized by staying within these arbitrary bounds. The exact values of the elements entering into the description of the complex gyrator circuit must, in any particular situation, be determined by the actual gain–bandwidth specification. It is, of course, separately necessary to satisfy the lower bound imposed on the aspect ratio of the resonator.

One engineering decision is to suppose that

$$\text{VSWR}(\min) = \sqrt{\text{VSWR}(\max)}. \tag{21}$$

If for engineering purposes the Q factor (Q_L) of the complex gyrator circuit is assumed approximately bounded by

$$1.9 \le Q_L(\text{eff}) \le 2.4$$

and if VSWR$(\max)$ is taken as

$$\text{VSWR}(\max) = 1.15$$

then the network problem indicates that the bounds on its normalized susceptance slope parameter (b') are approximately bracketed by

$$8 \le b' \le 16.$$

Once these two independent quantities are fixed, the normalized conductance (g) is given in the usual way by

$$Q_L = \frac{b'}{g}. \tag{22}$$

One possible adjustment technique for this class of device may be obtained by recognizing that the gyrator conductance at the midband frequency in a quarter-wave coupled device can be written as

$$\frac{y_t^2}{S(\max)} = \sqrt{3}b'\left(\frac{\omega_+ - \omega_-}{\omega_0}\right). \tag{23}$$

Scrutiny of this relationship suggests that in order to establish the required gyrotropy of the problem region, care must be taken to accurately realize both the admittance (y_t) of the transformer and the susceptance slope parameter (b').

Another feature which explains the sometimes unexpected success of commercial workers in this area is that while the frequency response of this class of circuit is akin to that of a degree-two filter circuit, no precise lower bound is usually in this instance imposed on its Q factor. An assurance that its upper bound is not violated is, therefore, sufficient for design.

If a normalized value of 8 is taken for the susceptance slope parameter by way of example, and if the substrate thickness is taken as 0.635 mm, then an end-fed triangular resonator is best at about 1.5 GHz, a disk one at 4.5 GHz, and a side-coupled triangular resonator at 13.5 GHz.

VII. Synthesis Procedure

If the substrate thickness forms part of the specification then the initial synthesis procedure adopted in this paper starts by evaluating the susceptance slope parameter of disk and triangular resonators based on the modified weakly magnetized model from a statement of the frequency and substrate thickness and its dielectric constant. In the example studied here,

$$f_0 = 3950\,\text{MHz}$$
$$H = 0.635\,\text{mm}.$$

Since both the susceptance slope parameter and the Q factor of the complex gyrator circuit are in the modified weakly magnetized model of the junction dependent upon the effective gyrotropy, the latter quantity must be fixed before proceeding with design. As the gain–bandwidth product of the specification is the paramount quantity, it determines in practice the gyrotropy The value for the triangular geometry is adopted here

$$(\kappa)_{\text{eff}} = 0.35.$$

It is separately recalled that the gyrotropy necessary to establish a given gain–bandwidth product is somewhat lower in the case of a junction using a disk geometry than in that employing a triangular one.

The effective permeabilities associated with the effective gyrotropy of the circuit are

$$\mu_e(f) = 0.88$$
$$\mu_{e,\text{eff}}(f) = 0.90.$$

Since the effective constitutive parameter cannot be calculated until r/H or R/H is available, a trial calculation based on the ideal magnetic-wall model of the problem region must be employed, in the first instance, to obtain

$$q_m = 0.87.$$

In order to be able to compare the values of the normalized susceptance slope parameter of the three weakly magnetized resonator shapes under consideration, it is assumed in the first place that all three designs are based on the same gyrotropy. The required values are then obtained by scaling the susceptance slope parameters obtained in Tables I–III by having recourse to (7) and (8)

$\mu_{e,\,\mathrm{eff}}(f)b' = 1.90$ apex coupled triangular resonator
$\mu_{e,\,\mathrm{eff}}(f)b' = 5.71$ disk resonator
$\mu_{e,\,\mathrm{eff}}(f)b' = 17.11$ side coupled triangular resonator.

A realizable specification which coincides with each resonator shape may now be obtained once the Q factor of each junction is evaluated in terms of its gyrotropy. The effective calculated value for the triangular resonator is

$$Q_L(\mathrm{eff}) = 2.42.$$

The network problem now indicates that the side-coupled triangular resonator is perhaps the most practical solution at this frequency. One possibility which is compatible with the calculated value of b' is

$$S(\min) = 1.075$$
$$S(\max) = 1.15$$
$$2\delta_0 = 0.22.$$

The equivalent gyrator circuit for this specification is defined by

$$b' = 17.22$$
$$g = 7.12$$
$$y_t = 2.86.$$

If there is no restriction on the thickness of the substrate, then the design can proceed on the basis of a specification as long as the gain–bandwidth product can be realized by the gyrator model under consideration. One possible specification is given by

$$S(\min) = 1.05$$
$$S(\max) = 1.15$$
$$2\delta_0 = 0.26$$

for which the equivalent gyrator circuit is described by

$$b' = 12.0$$
$$g = 6.0$$
$$Q_L(\mathrm{eff}) = 2.0$$
$$y_t = 2.63.$$

This specification cannot be realized by a side-coupled triangular resonator, but it can if a disk one on a substrate thickness of 0.345 mm is employed. The required effective gyrotropy is

$$(\kappa)_{\mathrm{eff}} = 0.28.$$

This calculation as well as that of the previous example also suggests that small variations in the Q factor of the specifications can produces large variations in the overall frequency response and in the details of the complex gyrator circuit.

VIII. Commercial Practice

The purpose of this section is to review the design and performance of one commercial design using a triangular resonator at 5 GHz and two designs in the open literature using disk ones at 4 and 9 GHz, respectively. While the details of each adjustment (in the absence of a Smith Chart display) are not sufficiently robust to draw firm conclusions about any agreement between design and practice, it does draw attention to the wide disparities met in the choices of the design parameters of practical circulators.

The frequency response of one 5-GHz quarter-wave coupled commercial device using a side-coupled triangle on a 1.90-mm substrate is indicated in Fig. 12. Its performance is specified by $S(\max) \approx 1.16$, $S(\min) \approx 1.04$, and $2\delta_0 \approx 0.21$. If an idealized template is superimposed on this data, then its normalized gyrator circuit is defined by $b' = 23.98$, $g = 9.137$, and $Q_L(\mathrm{eff}) = 2.625$. The physical gyrotropy κ employed in this assembly is equal to 0.56, the magnetic filling factor q_m is equal to 0.73, and the normalized direct magnetic field $\mu_0 H_0/M_0$ is of the order of 1.20. Its effective gyrotropy is, therefore, outside that of the weakly magnetized model considered here. This arrangement may, therefore, be described as a moderately rather than a very weakly or weakly magnetized arrangement. Its aspect ratio ($r/H = 1.44$) is also somewhat outside the value necessary to accurately reproduce the boundary conditions of the problem region. It is also of note that the value of susceptance slope parameter associated with this result is not at first sight compatible with the calculations undertaken here.

A quarter-wave coupled 4-GHz circulator using a disk gyromagnetic resonator on a 0.625–mm substrate, which was initially designed on the basis of a weakly magnetized solution, has also been separately described in [7]. Its performance is defined by $S(\max) \approx 1.20$, $S(\min) \approx 1.15$, and $2\delta_0 \approx 0.29$. If an idealized template is superimposed on this data then its complex gyrator circuit may be described by $b' = 5.97$, $g = 3.39$, and $Q_L(\mathrm{eff}) = 1.76$. The nominal gyrotropy (κ) of the material used in this design was equal to 0.53, its effective gyrotropy $(\kappa)_{\mathrm{eff}}$ is again estimated at about 0.40. The normalized direct magnetic field ($\mu_0 H_0/M_0$) was equal to about 1.0. The aspect ratio of the resonator is estimated as $R/H \approx 6.0$. Notwithstanding that the gyrotropy used here is about the same as that employed in connection with the description of the side-coupled triangular resonator, its gain–bandwidth product, in keeping with the use of a disk rather than a triangular resonator, is larger. It again goes to show that small differences in the Q factor of this sort of circuit

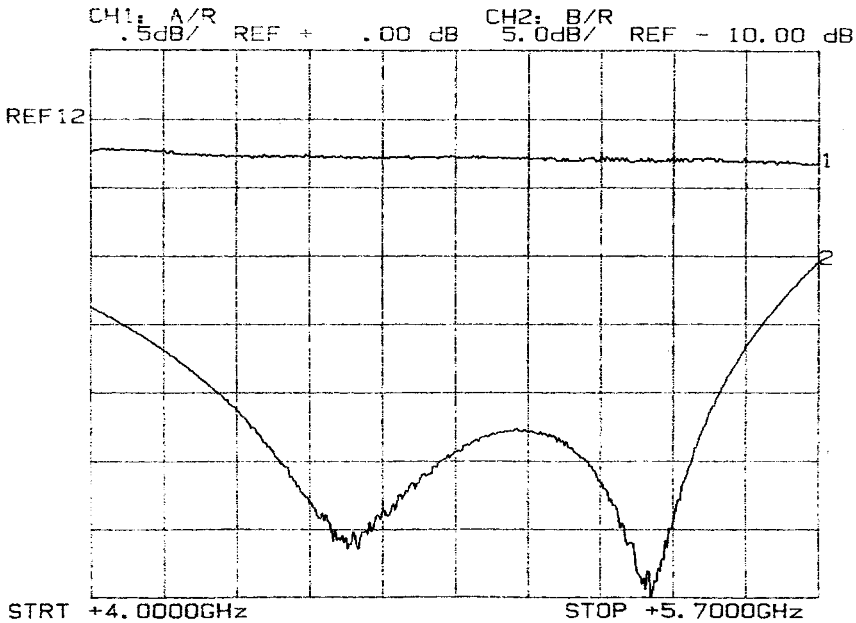


Fig. 12. Frequency response of quarter-wave coupled junction circulator using side-coupled triangular resonator ($r/H = 1.44$) (Courtesy W. T. Nisbet, RACAL-MESL).

can produce significant differences in the gain–bandwidth of the specification. While the design of this circulator initially assumed a very weakly magnetized model for its resonator, its gyrator circuit suggests that it is actually moderately magnetized. An empirical factor of two in the definition of its characteristic impedance was introduced in order to reconcile theory and experiment [7].

Another quarter-wave device using a disk resonator printed on a 0.635-mm substrate which has been described in the open literature at 9 GHz [40] is also worthy of review. Its performance is defined by $S(\max) \approx 1.15$, $S(\min) \approx 1.10$, and $2\delta_0 \approx 0.26$. If an idealized template is superimposed on this data then $b' = 7.80$, $g = 4.24$, and $Q_L(\text{eff}) = 1.84$. The nominal gyrotropy of the material in this design is 0.54, its effective value is about 0.438. The direct magnetic field ($\mu_0 H_0/M_0$) is 1.43. The aspect ratio of the resonator (R/H) is $R/H \approx 3.0$.

IX. Conclusions

One family of solutions that is fairly attractive for use in the construction of microstrip devices is one based on a weakly magnetized resonator in which the resonator shape is used in the synthesis problem. Some tradeoffs between the resonator shape, substrate thickness, maximum and minimum ripple levels of the network problem and the gyrotropy of the problem region have been examined in this paper. The details of a commercial version using a triangular resonator and two in the open literature employing disk ones, which may be deemed outside the weakly magnetized model outlined here, have been separately reviewed. While this paper does not presume to fully solve all aspects entering into the design procedure of this class of device, it does suggest that where possible, a design based on a weakly rather than a very weakly magnetized model has merit. The family of solutions outlined here is the only one possible at frequencies above about 50 GHz.

References

[1] H. Hershenov, "X-band microstrip circulator," *Proc. IEEE*, vol. 54, pp. 2022–2023, Dec. 1966.

[2] ——, "All garnet substrate microstrip circulators," *Proc. IEEE*, vol. 55, pp. 696–697, May 1967.

[3] D. Masse, "Broadband microstrip junction circulators," *Proc. IEEE*, vol. 56, pp. 352–353, Mar. 1968.

[4] P. Barsony, "Some problems of microstrip circulators," presented at the Annu. Res. Inst. Telecommun., Budapest, Hungary, 1974.

[5] C. H. Oxley, T. J. Brazil, J. J. Purcell, and R. Genmer, "Design and performance of J band ferrite microstrip circulators," *Proc. Inst. Elect. Eng.*, vol. 125, no. 8, Aug. 1978.

[6] J. Helszajn and M. Powesland, "Low-loss high power microstrip circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 572–578, July 1981.

[7] N. Krause "Improved microstrip circulator design by resonance components," in *Proc. EuMC*, Amsterdam, The Netherlands, 1981, pp. 383–388.

[8] C. E. Fay and R. L. Comstock, "Operation of the ferrite junction circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 15–27, Jan. 1965.

[9] H. Bosma, "On stripline Y-circulation at UHF," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 61–72, Jan 1964.

[10] J. Helszajn, "Quarter-wave coupled junction circulators using weakly magnetized disk resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 800–806, May 1982.

[11] J. Helszajn, D. S. James, and W. T. Nisbet, "Circulators using planar triangular resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 188–193, Feb. 1979.

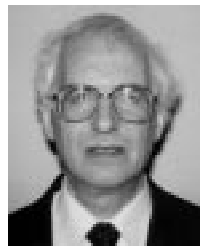
[12] J. Helszajn and R. W. Lyon, "Mode charts of magnetized and demagnetized planar hexagonal resonators on ferrite substrates using finite elements," *Proc. Inst. Elect. Eng. Microwaves, Antennas, Propagat.*, vol 131, pp. 420–422, Dec. 1984.

[13] L. K. Anderson, "An analysis of broadband circulators with external tuning elements," *IEEE Trans. Microwave Theory Technol.*, vol. MTT-15, pp. 42–47. Jan. 1967.

[14] J. Helszajn, "Synthesis of quarter-wave coupled circulators with Chebyshev characteristics," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 764–769, 1972.

[15] R. Levy and J. Helszajn, "Specific equations for one and two section quarter-wave matching networks for stub resistor loads," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 56–62, Jan. 1982.

[16] ——, "Synthesis of short line matching networks for resonant loads," *Proc. Inst. Elect. Eng.*, vol. 130, pt. H, pp. 385–390, 1983.
[17] J. Helszajn, "Scattering matrix of junction circulator with Chebyshev characteristics," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 548–554, 1975.
[18] J. Helszajn, R. Baars, and W. T. Nisbet, "Characteristics of circulators using planar triangular and disk resonators symmetrically loaded with magnetic walls," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 616–662, June 1980.
[19] J. Helszajn and W. T. Nisbet, "Circulators using planar Wye resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 689–699, July 1981.
[20] J. Helszajn, "Standing wave solution of planar irregular and hexagonal resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 562–567, June 1981.
[21] J. Helszajn and C. Bradley, "Experimental characterization of reciprocal planar microstrip three-port junctions," *Proc., Inst. Elect. Eng. Microwaves, Antennas, Propagat.*, vol. 138, pp. 91–97, Feb. 1991.
[22] J. Helszajn and R. Baars, "Synthesis of wideband planar circulators using undersized disk resonators," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1681–1687, Oct. 1991.
[23] J. B. Davies and P. Cohen, "Theoretical design of symmetrical junction stripline circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 506–512, Nov. 1963.
[24] W. Simon, "Broadband strip-transmission line Y-junction circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 335–345, May 1965.
[25] Y. S. Wu and F. J. Rosebaum, "Wideband operation of microstrip circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 849–856, Oct. 1974.
[26] J. Helszajn, "Operation of tracking circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 700–712, July 1981.
[27] T. Miyoshi, S. Yamaguchi, and S. Goto, "Ferrite planar circuits in microwave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 595–600, July 1997.
[28] T. Miyoshi and S. Miyauchi, "The design of planar circulators for wideband operation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 210–214, Mar. 1980.
[29] Y. Ayasli, "Analysis of wide-band stripline circulators by integral equation technique," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 200–209, Mar. 1980.
[30] I. Wolff and N. Knoppik, "Rectangular and circular disk capacitors and resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 857–864, Oct. 1974.
[31] G. Kompa, "S-matrix computation of microstrip discontinuities," *AEU*, vol. 30, pp. 58–64, 1976.
[32] W. T. Nisbet and J. Helszajn, "Mode charts for microstrip resonators using a transverse resonance method," *Inst. Elect. Eng., Microwaves, Opt., Acoust.*, vol. 3, pp. 69–77, Mar. 1979.
[33] ——, "Microstrip synthesized from waveguide equivalent," *MSN*, pp. 117–122, Jan. 1981.
[34] J. Helszajn, "Microwave measurement techniques for below resonance junction circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 347–351, May 1973.
[35] G. Riblet, "Measurement of the equivalent admittance of three-port circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 401–405, May 1977.
[36] Y. Akaiwa, "Correction to mode classification of a triangular ferrite post for Y-circulator operation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, p. 709, July 1979.
[37] J. Helszajn, "Contour integral equation formulation of complex gyrator admittance of junction circulators using triangular resonators," *Proc. Inst. Elect. Eng. Microwaves, Antennas, Propagat.*, vol. 132, pp. 255–260, July 1985.
[38] R. W. Lyon and J. Helszajn, "A finite element analysis of planar circulators using arbitrarily shaped resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1964–1974, Nov. 1982
[39] H. Schlömann, "Microwave behavior of partially magnetized ferrites," *J. Appl. Phys.*, vol. 41, pp. 204–214, 1970.
[40] S. A. Ivanov and P. Dunkov, "X-band microstrip circulator on a ferrite substrate," in *Proc. VIII Microcool, Coll. Microwave Commun.*, Budapest, Hungary, Aug. 25–29, 1986, pp. 353–354.



Joseph Helszajn (M'64–SM'87–F'92) received the M.S.E.E. degree in electrical engineering from the University of Santa Clara, Santa Clara, CA, the Ph.D. degree from Leeds University, Leeds, U.K., in 1969 for his work on spinwave instabilities in magnetic insulators at large RF signal level, the D.Sc. degree for his early collected works on gyromagnetic devices and circuits, and the D.Eng. degree from Heriot-Watt University, Edinburgh, Scotland, in 1974 and 1995, respectively.

He is an international authority on nonreciprocal microwave circuits and devices. He gained his first qualification at what is currently the University of North London and then undertook National Service in the Royal Air Force (1955 to 1957). He acquired his early industrial experience on the east and west coasts of the United States. He joined the Department of Electrical Engineering, Heriot-Watt University, in 1971, where he was instrumental in laying the foundation of what is now its Microwave Laboratory. He was appointed to a Personal Chair in the Department of Microwave Engineering in 1982, and held the title of Distinguished Visiting Professor at Arizona State University during part of the 1987 academic year. He has authored ten engineering text books which have unified the important nonreciprocal branch of microwave engineering: *Principles of Microwave Ferrite Engineering* (New York: Wiley, 1969), *Non-reciprocal Microwave Junctions & Circulators*, (New York: Wiley, 1975), *Passive and Active Microwave Circuits*, (New York: Wiley, 1978, 1980), *YIG Resonators and Filters*, (New York: Wiley, 1985) *Ferrite Phase Shifters and Control Devices*, (New York: McGraw-Hill, 1989), *Synthesis of Lumped Element, Distributed and Planar Filter Circuits*, (New York: McGraw-Hill, 1990), *Microwave Engineering; Passive, Active and Non-Reciprocal Circuits*, (New York: McGraw-Hill, 1992, 1993) *Microwave Passive Planar Circuits and Filters*, (New York: Wiley, 1993, 1994), *Green's Function, Finite Elements and Microwave Planar Circuits*, (New York: Wiley, 1996), *Theory and Practice of Waveguide Junction Circulators*, (New York: Wiley, 1998). He was one of two honorary editors (1981 and 1997) of the *Proceedings of the Institution of Electrical Engineers on Microwaves, Antennas, and Propagation*.

Prof. Helszajn is a Chartered Engineer (CEng.), a Fellow of the Institution of Electrical Engineers (FIEE), the Institute of Electrical and Electronic Engineers (FIEEE), the Royal Society of the Arts (FRSA), the City and Guilds Institute (FGCI), the Royal Society of Edinburgh (FRSE), and the Royal Academy of Engineering (FEng). He is a member of the editorial board of the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES. He was the 1995 recipient of the Institution of Electrical Engineers J. J. Thomson Medal, and was appointed an Officer of the Order of the British Empire (OBE) in the 1997 Queen's Birthday Honors List.